NATIONAL UNIVERSITY OF SINGAPORE

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS

SEMESTER 2 EXAMINATION 2013–2014

MA1301 Introductory Mathematics

May 07, 2014 Time allowed: 2 hours

INSTRUCTIONS TO STUDENTS

1. This examination paper contains a total of **SEVEN (7)** questions and comprises **FOUR (4)** printed pages.
2. Answer **ALL** questions. Write your solutions in the **ANSWER BOOK**.
3. Please start each question on a new page.
4. Please write your student number only. Do **NOT** write your name on the answer book.
5. Total marks for this exam is **60**. The marks for each question are indicated at the beginning of each question.
6. This is a **CLOSED BOOK** examination. **Two A4-sized helpsheet (two-sided) are allowed.**
7. Candidates may use **scientific calculators or graphing calculators**. However, they should lay out systematically the various steps in the calculations.

Question 1 [8 marks]

(i) (4 marks) Find the domain and range of the functions:

$$f : \{x \in \mathbf{Z}, |x| \geq 3\} \to \mathbf{R}, f(x) = x + 1$$

and

$$g(x) = \sqrt{x^4 - 16}.$$

(ii) (1 mark) Explain why $g \circ f$ exists.

(iii) (3 marks) Find the smallest integer c such that for any $x \in D_f$ and $x \geq c$, $g \circ f$ has an inverse (function).

Question 2 [8 marks]

(i) (4 marks) Express

$$\frac{1}{k(k+1)(k+2)}$$

as partial fractions by considering

$$\frac{1}{k(k+1)(k+2)} = \frac{A}{k} + \frac{B}{k+1} + \frac{C}{k+2}.$$

(ii) (4 marks) Find

$$\sum_{k=1}^{n} \frac{1}{k(k+1)(k+2)}$$

and express your answer in n.

Question 3 [9 marks]

(i) (3 marks) Verify the differentiation rule:

$$(f(x)g(x)h(x))' = f'(x)g(x)h(x) + f(x)g'(x)h(x) + f(x)g(x)h'(x).$$

(ii) (6 marks) Prove the generalized product rule by induction: if $n \geq 2$ and $f_1(x), f_2(x) \cdots, f_n(x)$ are differentiable functions on x, then

$$\frac{d(f_1(x)f_2(x)\cdots f_n(x))}{dx} = \sum_{i=1}^{n} \frac{d(f_i(x))}{dx} f_1(x)\cdots f_{i-1}(x)f_{i+1}(x)\cdots f_n(x).$$

Question 4 [9 marks]

Let Π be the plane containing the points $A(0,1,0)$, $B(1,-3,1)$ and $C(2,-1,-1)$ and l be the line passing through the point $(x,1,0)$ in the direction $(0,1,-2)$.

(i) (2 marks) Find the equation of the plane Π.

(ii) (5 marks) Suppose the distance between l and the point $P(1,2,3)$ is 3. Determine the equation of the line l.

(iii) (2 marks) Find the acute angle between the line l and the plane Π.

Question 5 [10 marks]

Consider the following differential equation

$$xy\frac{dy}{dx} + 4x^2 + y^2 = 0$$

with $y(2) = -7, x > 0$.

(i) (2 marks) Prove that the equation can be changed to

$$\frac{y}{x}\frac{dy}{dx} = -4 - (\frac{y}{x})^2.$$

(ii) (2 marks) Let $z = \frac{y}{x}$. Express

$$\frac{dy}{dx}$$

in terms of x, z.

(iii) (6 marks) Solve the differential equation.

Question 6 [8 marks]

(i) (2 marks) Find the intersection points of the two curves $y = x^{1/3}$ and $y = \frac{x}{4}$.

(ii) (6 marks) Determine the volume of the solid obtained by rotating the portion of the region bounded by $y = x^{1/3}$ and $y = \frac{x}{4}$ that lies in the first quadrant **about the y-axis**.

Question 7 [8 marks]

The number t is real and not an integer multiple of $\pi/2$. The complex numbers x_1, x_2 are the roots of the equation

$$\tan^2(t)x^2 + \tan(t)x + 1 = 0.$$

(i) (4 marks) Show that

$$x_1 = (\cos(2\pi/3) + i\sin(2\pi/3))\cot(t)$$

and

$$x_2 = (\cos(2\pi/3) - i\sin(2\pi/3))\cot(t).$$

(ii) (4 marks) Find $x_1^n + x_2^n$. Express your answer in t and n.

End of paper.