NATIONAL UNIVERSITY OF SINGAPORE

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS

SEMESTER 1 EXAMINATION 2014–2015

MA1301 Introductory Mathematics

November 25, 2014 Time allowed: 2 hours

INSTRUCTIONS TO STUDENTS

1. This examination paper contains a total of **EIGHT (8)** questions and comprises **THREE (3)** printed pages.

2. Answer **ALL** questions. Write your solutions in the **ANSWER BOOK**.

3. Please start each question on a new page.

4. Please write your student number only. Do **NOT** write your name on the answer book.

5. Total marks for this exam is **60**. The marks for each question are indicated at the beginning of each question.

6. This is a **CLOSED BOOK** examination. **Two A4-sized helpsheet (two-sided) are allowed.**

7. Candidates may use **scientific calculators or graphing calculators**. However, they should lay out systematically the various steps in the calculations.

Question 1 [5 marks]

(i) (4 marks) Find the domain and range of the functions:

$$f : \{x \in \mathbf{Z}, |x| \geq 3\} \to \mathbf{R}, f(x) = x + 2$$

and

$$g(x) = \ln(x^6 - 1/64).$$

(ii) (1 mark) Explain why $g \circ f$ exists.

Question 2 [8 marks]

Solve the inequality

$$\frac{4^{x+1} - 5(2^{x+2})}{2(2^{x-1} - 1)} \geq -8$$

by making use of the substitution $y = 2^x$.

Question 3 [10 marks]

(i) (5 marks) Find the number a such that the two curves $y = ax^2$ and $y = \ln x$ have the same tangent line at an intersection point.

(ii) (5 marks) Suppose

$$y = \frac{2x + 1}{x + 1}.$$

Prove by induction:

$$\frac{d^n y}{dx^n} = \frac{(-1)^{n-1} n!}{(x + 1)^{n+1}}, n \geq 2.$$

Question 4 [8 marks]

Let l be the intersection line of two planes Π_1 and Π_2 given by

$$\Pi_1 : x - y - z = -1, \qquad \Pi_2 : x - y + z = 3.$$

Let Π be the plane passing through the point $P(2, 1, 2)$ and perpendicular to l. Find the equation of the plane Π.

Question 5 [7 marks]

Solve the differential equation

$$xy^2 \frac{dy}{dx} = x^3 + y^3$$

by making use of the substitution

$$u = \frac{y}{x}.$$

Question 6 [8 marks]

Suppose a is a fixed positive real number and $m, n > 0$ are two fixed constant natural numbers. If two positive real numbers x, y satisfy $x + y = a$, find the largest possible value of $x^m y^n$.

Question 7 [8 marks]

Suppose a and b are two fixed constant real numbers. Determine the volume of the solid obtained by rotating the portion of the region bounded by

$$y = b(\frac{x}{a})^2 \text{ and } y = b\frac{|x|}{|a|}$$

about the x-axis.

Question 8 [6 marks]

Determine whether the series

$$1 + (\frac{1}{2} + \frac{1}{2}i) + (\frac{1}{2} + \frac{1}{2}i)^2 + (\frac{1}{2} + \frac{1}{2}i)^3 \cdots$$

has sum to infinity. If yes, find the sum; if no, justify your answer.

End of paper.