NATIONAL UNIVERSITY OF SINGAPORE

MA1521 — CALCULUS FOR COMPUTING

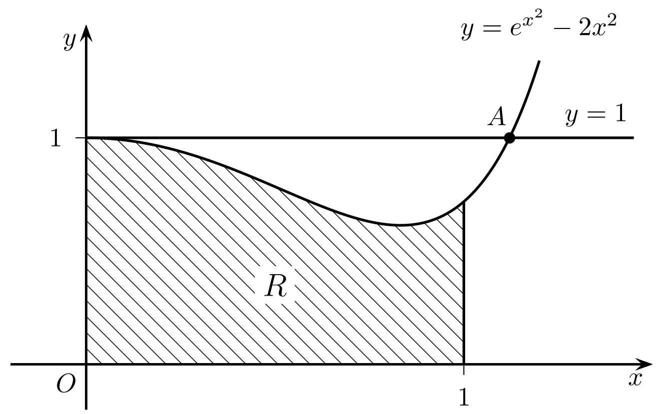
(Semester 1 : AY2014/2015)

Time allowed : 2 hours

INSTRUCTIONS TO CANDIDATES

1. Please write your matriculation/student number only. Do not write your name.
2. This examination paper contains a total of **SIX (6)** questions and comprises **FOUR (4)** printed pages.
3. This examination carries a total of 60 marks.
4. Answer **ALL** questions.
5. Please start each question on a new page.
6. This is a **CLOSED BOOK** examination.
7. Your are allowed to use four A4-size, double-sided help sheets.
8. You may use calculators. However, you should lay out systematically the various steps in the calculations.

Question 1 [10 marks]



The above diagram shows part of a curve whose equation is

$$y = e^{x^2} - 2x^2, \quad x \geq 0.$$

The line $y = 1$ meets the curve at the point A. The region R is bounded by the axes, the line $x = 1$ and the curve.

(i) Let α be the x-coordinate of A. Find the integer n for which

$$1 + \frac{n-1}{10} < \alpha < 1 + \frac{n}{10}.$$

(ii) Use the Newton-Raphson method to find α. Give your answer to three significant figures.

(iii) Use the trapezoidal rule with three ordinates to approximate the area of the region R. Give your answer to four significant figures.

(iv) The region R is rotated completely about the y-axis. Find the **exact** volume of the generated solid.

Question 2 **[10 marks]**

Let

$$f(x,y) = x(x^2+3y^2) - 6(x^2+y^2) + 7.$$

P is the point on the surface $z = f(x,y)$ where $x = 0$ and $y = -1$.

(i) Find a Cartesian equation of the tangent plane at the point P.

(ii) Find the value of c for which the directional derivative of f at the point P in the direction of $c\mathbf{i} + \mathbf{j}$ is 0.

(iii) Find the x- and y-coordinates of all critical points of f, and determine the nature of each critical point.

(iv) Find the **exact** value of $g'(0)$, given that $g(z) = \displaystyle\int_{e^{3z}}^{1} e^{f(1,y)+2z+1}\, dy$.

Question 3 **[10 marks]**

Let $f(x) = \dfrac{4x-5}{2x^2-4x+5}$.

(i) Determine the constants A and B such that

$$2x^2 - 4x + 5 \equiv A + B(x-1)^2.$$

(ii) Obtain the Taylor series expansion for the function f at $x = 1$.

(iii) Use the result in (ii) to find the **exact** value of $f^{(2014)}(1)$.

(iv) Use the result in (ii) to find the **exact** value of $g^{(1521)}(1)$, where $g(x) = f(2x-1)$.

Question 4 **[10 marks]**

Find the general solution of the following differential equations.

(a) $\dfrac{dy}{dx} = y\cot x + 3\csc x, \quad 0 < x < \pi.$

(b) $\dfrac{d^2y}{dx^2} - 6\dfrac{dy}{dx} + 9y = e^{3x}\ln x.$

Question 5 [10 marks]

Determine whether each of the following series is convergent or divergent. Justify your answers.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n n^4}{4+n^5}$.

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n^{3n}}{(3n+1)!}$.

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos\frac{1}{n} + \cos\frac{2}{n} + \cdots + \cos\frac{n}{n}}{n^2}$.

Question 6 [10 marks]

(a) Let $I_n = \displaystyle\int_0^{\pi/4} \tan^{2n} x \, dx$, $n \geq 0$.

(i) Show that for $n \geq 1$,

$$I_n + I_{n-1} = \frac{1}{2n-1}.$$

(ii) Use the result in (i) to find the **exact** value of

$$\int_0^{\pi/4} \tan^4 x \, dx.$$

(iii) Let $J_n = (-1)^n I_n$, $n \geq 0$. Show that for $n \geq 1$,

$$J_n - J_{n-1} = (-1)^n \frac{1}{2n-1}.$$

Hence, show that for any positive integer N,

$$I_N = (-1)^N \left(\sum_{n=1}^{N} \frac{(-1)^n}{2n-1} + \frac{\pi}{4} \right).$$

(b) Find the **exact** value of

$$\sum_{n=2}^{\infty} \frac{(2n+1)\, 2^n}{(n-1)!}.$$

END OF PAPER