NATIONAL UNIVERSITY OF SINGAPORE

MA1521 — CALCULUS FOR COMPUTING

SEMESTER 2: AY 2014/15

Time allowed : 2 hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains a total of **SEVEN (7)** questions and comprises **FOUR (4)** printed pages.
2. Answer **ALL** questions. This examination carries a total of **100** marks. The marks for questions are not necessarily the same; marks for each question are indicated at the beginning of the question.
3. Use a separate page for each question.
4. This is a **CLOSED BOOK** examination.
5. Each candidate may bring two pieces of A4-size, double-side, handwritten formula sheet.
6. Candidates may use calculators. However, they should lay out systematically the various steps in the calculations.

Question 1 [13 marks]

Let $f(x,y) = (x^2+3x+3)e^x \cdot (y^2+3y+3)e^y$.

(i) Find all the critical points of f.

(ii) Determine if f has a local maximum, local minimum or saddle point at each of the critical points. Justify your answers.

Question 2 [10 marks]

A delivery company accepts only rectangular boxes the sum of whose length x and girth (perimeter of a cross-section $2y+2z$) does not exceed 270 cm. Using the Lagrange multipliers method, find the dimensions of an acceptable box of largest volume. [You may assume that the largest volume exists without proof.]

Question 3 [14 marks]

Find the following indefinite integrals.

(a) $\displaystyle\int \sin(\ln x)\,dx$.

(b) $\displaystyle\int \frac{1}{x^{2015}-x}\,dx$.

Question 4 [10 marks]

Determine if each of the series is convergent or divergent. Justify your answers.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln(n^2+n^3)}{\ln(2^n+3^n)}$.

(b) $\displaystyle\sum_{n=1}^{\infty} \left(\tan\frac{1}{n} - \sin\frac{1}{n}\right)$.

Question 5 [17 marks]

For each $n \geq 0$, define

$$I(n) = \int_2^\infty \frac{1}{(x^2-1)^n}\,dx.$$

(i) Show that $I(2) = \dfrac{1}{3} - \dfrac{1}{4}\ln 3$.

(ii) Show that for $n \geq 2$,

$$(2n-3)I(n-1) + (2n-2)I(n) = \frac{2}{3^{n-1}}.$$

(iii) Using (i) and (ii), evaluate $I(3)$ and $I(4)$.

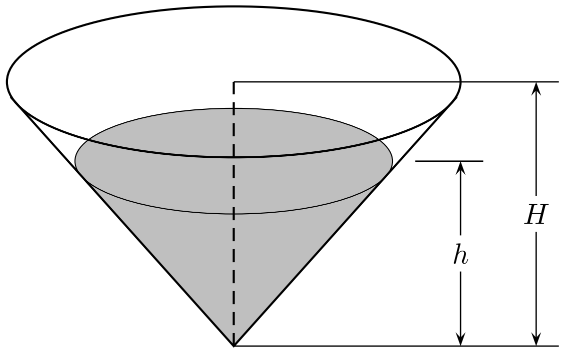
Question 6 [18 marks]

(a) A cone-shaped water tank is shown below. When the tank is full, a valve is opened at the bottom of the tank. The depth of the water is halved after 1 hour.

It is given that the depth of the water h and the time t can be modeled by

$$\frac{dh}{dt} = -\frac{c}{h^{3/2}},$$

for some constant $c > 0$. How long will it take for the tank to drain completely?



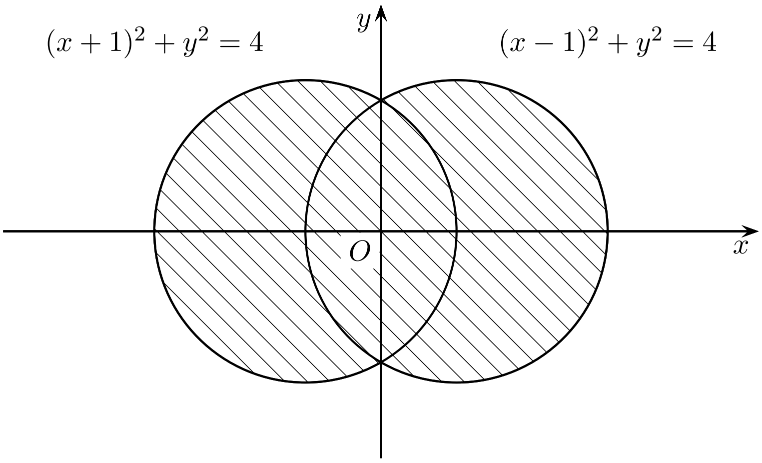
(b) Solve the following ordinary differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = -e^{-x},$$

where $y = 1$ and $\dfrac{dy}{dx} = 1$ at $x = 0$.

Question 7 [18 marks]

(a) Let R be the shaded region below:



(i) Find the volume of the solid formed by rotating R about the x-axis.

(ii) Find the volume of the solid formed by rotating R about the y-axis.

(b) Fine the arc length of the following curve

$$y = \sin^{-1} x + \sqrt{1 - x^2}.$$

End of Paper