NANYANG TECHNOLOGICAL UNIVERSITY
SEMESTER 1 EXAMINATION 2014–2015
MH1811 – MATHEMATICS 2

DECEMBER 2014 TIME ALLOWED: 2 HOURS

Matriculation Number:

Seat Number:

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **SIX (6)** questions and comprises **SEVENTEEN (17)** pages, including an Appendix.
2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.
3. This **IS NOT** an **OPEN BOOK** exam. However, a list of formulae is provided in the Appendix.
4. Candidates may use calculators. However, they should write down systematically the steps in the workings.
5. All your solutions should be written in this booklet within the space provided after each question. If you use an additional answer book, attach it to this booklet and hand them in at the end of the examination.

For examiners only

Question	Marks
1 (15)	
2 (15)	
3 (20)	

Question	Marks
4 (20)	
5 (15)	
6 (15)	

TOTAL (100)

QUESTION 1. **(15 Marks)**

Consider the second order differential equation,

$$y'' - 2y' + 5y = 0. - - - - (*)$$

(a) Find the general solution to the differential equation (*).

(b) Solve the given initial-value problem

$$y'' - 2y' + 5y = 0, y(\pi) = 0, y'(\pi) = 2.$$

Question 1 continues on page 3.

(c) Find the general solution to the non-homogeneous differential equation,

$$y'' - 2y' + 5y = e^{2x}.$$

End of Question 1.

QUESTION 2. **(15 Marks)**

(a) Let $f(x) = \cos x + \ln(1 + x + 3x^2)$.

(i) Find the Taylor polynomial $T_2(x)$ up to degree 2 of f at $x = 0$.

(ii) Use your answer in part (i) to estimate the value $f(-0.1)$.

Question 2 continues on page 5.

(b) At what point(s) on the paraboloid $x = y^2 + z^2$ is the tangent plane parallel to the plane $x + 2y + 3z = \pi$?

End of Question 2.

QUESTION 3. **(20 Marks)**

Consider the function $g(x, y) = \sqrt{x^2 + y^2 - 9}$.

(a) Determine the domain S of $g(x, y)$ and the level curve C that passes through the point $P(-5, 0)$.

(b) On the same sketch, indicate the domain S and the level curve C.

Question 3 continues on page 7.

(c) Find the rate of change of $g(x, y)$ at $Q(2, -3)$ along the direction $\mathbf{v} = -\mathbf{i} + \sqrt{3}\mathbf{j}$. Is $g(x, y)$ increasing or decreasing at Q?

(d) Suppose $x = (3s - t)e^{s-2t}$ and $y = st - 2$. Find the partial derivative $\dfrac{\partial g}{\partial s}$ at $s = 2$ and $t = 1$.

End of Question 3.

QUESTION 4. **(20 Marks)**

Consider the region D in $\mathbb{R}^2$ where

$$D = \{ (x, y) \mid 0 \leq y \leq 2x - x^2 \}.$$

(a) Sketch the region D on an (x, y)-plane.

(b) Find the volume of the solid under the surface $z = xy$ and above the region D.

Question 4 continues on page 9.

(c) Let $h(x, y) = xy^k$, where k is a positive integer. Find the coordinates of the point at which h is maximum on the boundary of D.

End of Question 4.

QUESTION 5. **(15 Marks)**

(a) Use the ratio test to determine the radius of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(3n+2)!x^n}{(2n-1)!(n!)}.$$

Question 5 continues on page 11.

(b) Is the series $\displaystyle\sum_{n=1}^{\infty} \frac{n^3}{3^n + \sqrt{n+1}}$ convergent? Justify your answer.

Question 5 continues on page 12.

(c) Is the series $\sum_{n=10}^{\infty} \frac{1}{n\left((3\ln n)^2+1\right)}$ convergent? Justify your answer.

End of Question 5.

QUESTION 6 **(15 Marks)**

(a) By setting $u = y^2$, solve the following differential equation

$$2xyy' + (x-1)y^2 = x^2e^x.$$

Question 6 continues on page 14.

(b) Consider the following differential equation

$$\left(\frac{y}{x^2} - \frac{2}{x}\right)\, dx + \left(\frac{1}{x} - \frac{1}{y}\right)\, dy = 0. - - - (**)$$

(i) Determine the values of the constants r and s such that $I(x, y) = x^r y^s$ is an integrating factor to transform the given differential equation (**) into an exact one.

Question 6 continues on page 15.

(ii) Proceed to solve the differential equation (**).

END OF PAPER

Appendix

Derivatives

$\frac{d}{dx}(x^n) = nx^{n-1}$

$\frac{d}{dx}(\sin x) = \cos x$

$\frac{d}{dx}(\cos x) = -\sin x$

$\frac{d}{dx}(\tan x) = \sec^2 x$

$\frac{d}{dx}(\cot x) = -\csc^2 x$

$\frac{d}{dx}(\sec x) = \sec x \tan x$

$\frac{d}{dx}(\csc x) = -\csc x \cot x$

$\frac{d}{dx}(e^x) = e^x$

$\frac{d}{dx}(\ln x) = \frac{1}{x}$

$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$

$\frac{d}{dx}(\sinh x) = \cosh x$

$\frac{d}{dx}(\cosh x) = \sinh x$

$\frac{d}{dx}(\tanh x) = \text{sech}^2 x$

$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{x^2+1}}$

Antiderivatives

$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C, \; n \neq -1$

$\int \frac{1}{x} \, dx = \ln|x| + C$

$\int \sin x \, dx = -\cos x + C$

$\int \cos x \, dx = \sin x + C$

$\int \sec^2 x \, dx = \tan x + C$

$\int \csc^2 x \, dx = -\cot x + C$

$\int \tan x \sec x \, dx = \sec x + C$

$\int \cot x \csc x \, dx = -\csc x + C$

$\int \tan x \, dx = \ln|\sec x| + C$

$\int e^x \, dx = e^x + C$

$\int \sec x \, dx = \ln|\sec x + \tan x| + C$

$\int \csc x \, dx = -\ln|\csc x + \cot x| + C$

$\int \frac{1}{\sqrt{1-x^2}} \, dx = \sin^{-1}(x) + C$

$\int \frac{1}{1+x^2} \, dx = \tan^{-1} x + C$

Trigonometry

$\sin^2\theta + \cos^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$

$\sin(2\theta) = 2\sin\theta\cos\theta$

$\cos(2\theta) = \cos^2\theta - \sin^2\theta$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

Taylor Series of $f(x)$ about a

$$f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n + \cdots$$

Maclaurin Series

$$\frac{1}{1-x} = 1 + x + x^2 + \cdots + x^n + \cdots \qquad (|x| < 1)$$

$$e^x = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + \cdots \qquad (|x| < \infty)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + \frac{(-1)^n x^{2n+1}}{(2n+1)!} + \cdots \qquad (|x| < \infty)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + \frac{(-1)^n x^{2n}}{(2n)!} + \cdots \qquad (|x| < \infty)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots + \frac{(-1)^{n+1} x^n}{n} + \cdots \qquad (-1 < x \leq 1)$$

Linearization of $f(x,y)$ about (a,b)

$$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$

MH1811 MATHEMATICS 2

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.