INTRODUCTORY STATISTICS

STATS101
SINGAPORE MANAGEMENT UNIVERSITY
Tutor: Ravins Cheung (A+)

Module 1: Introduction
1.Tabulating and graphing data distributions
2.Measures of dispersion
3.Measures of central tendency
4.Mean and variance for grouped/sample data
5.Summation Notation
6.Recap

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Tabulating and graphing data distributions

Objectives

- Helps to organize and present data
- Describes analysis of statistics

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Tabulating Categorical Data

Representing data that has been categorized in a table form.

30 people were randomly surveyed, and their responses were collated

Marital Status	Absolute Freq	Relative Freq
Single	13	0.325
Married	18	0.450
Others	9	0.225

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Tabulating Categorical Data

Test scores of 20 students were collated as below.

12	**13**	**17**	**21**	**24**	**24**	**26**	**27**	**27**	**30**
32	**35**	**37**	**38**	**41**	**43**	**44**	**46**	**53**	**58**

Class	Frequency	Relative Frequency	Cumulative Frequency
(10,20]	3	0.15	0.15
(20,30]	7	0.35	0.5
(30,40]	4	0.2	0.7
(40,50]	4	0.2	0.9
(50,60]	2	0.1	1

Note: (10; 20] refers to values between 10 (exclusive) and 20 (inclusive) etc.

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Graphing Data Distributions

Represents data (raw/tabulated) in graphs for ease of interpretation

Types of graphs

1. Pie Chart
2. Bar Chat
3. Side-by-side chart
4. Absolute frequency histogram
5. Relative frequency histogram
6. Cumulative frequency polygon
7. Stem and leaf diagram
8. Boxplots

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Graphing Data Distributions

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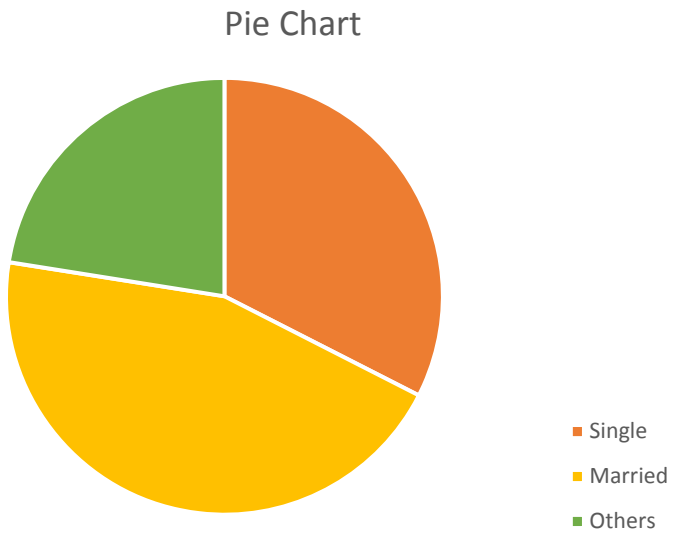

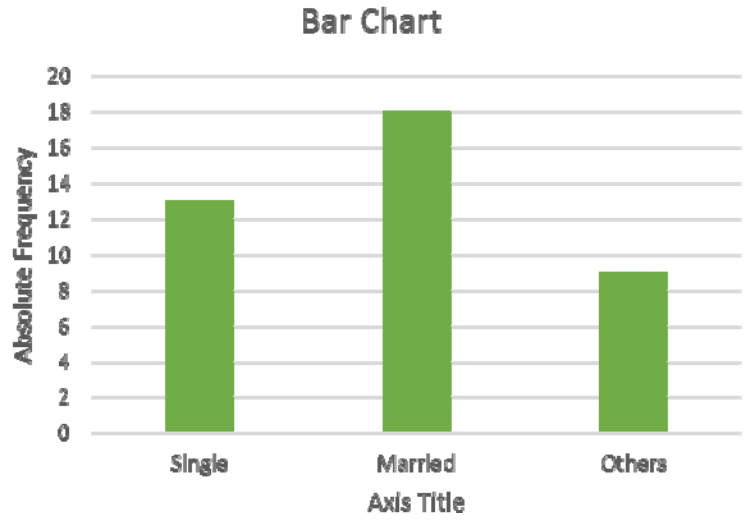

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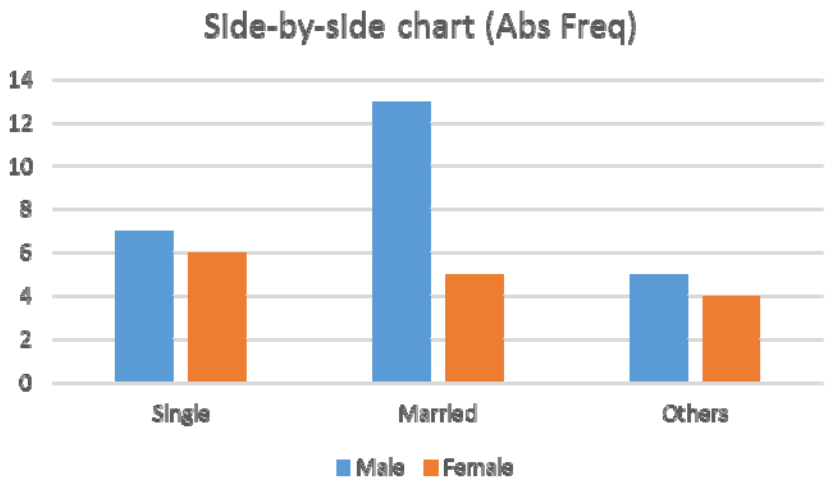
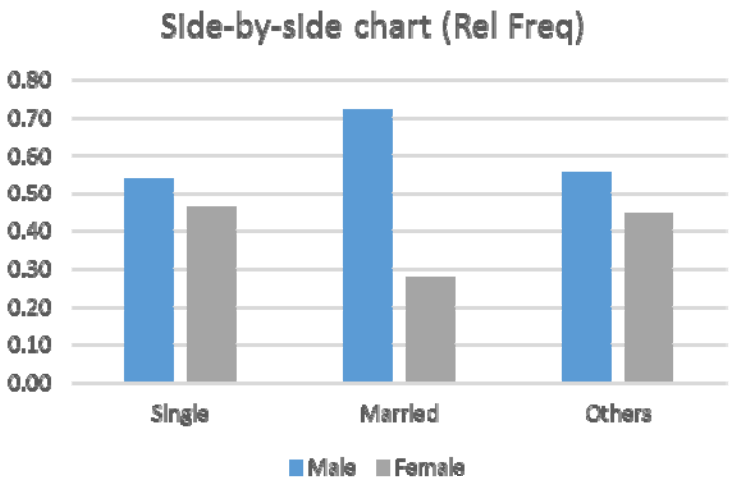
Graphing Data Distributions

Marital Status	Male	Female
Single	7	6
Married	13	5
Others	5	4

Marital Status	Male	Female
Single	0.54	0.46
Married	0.72	0.28
Others	0.56	0.44





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Graphing Data Distributions

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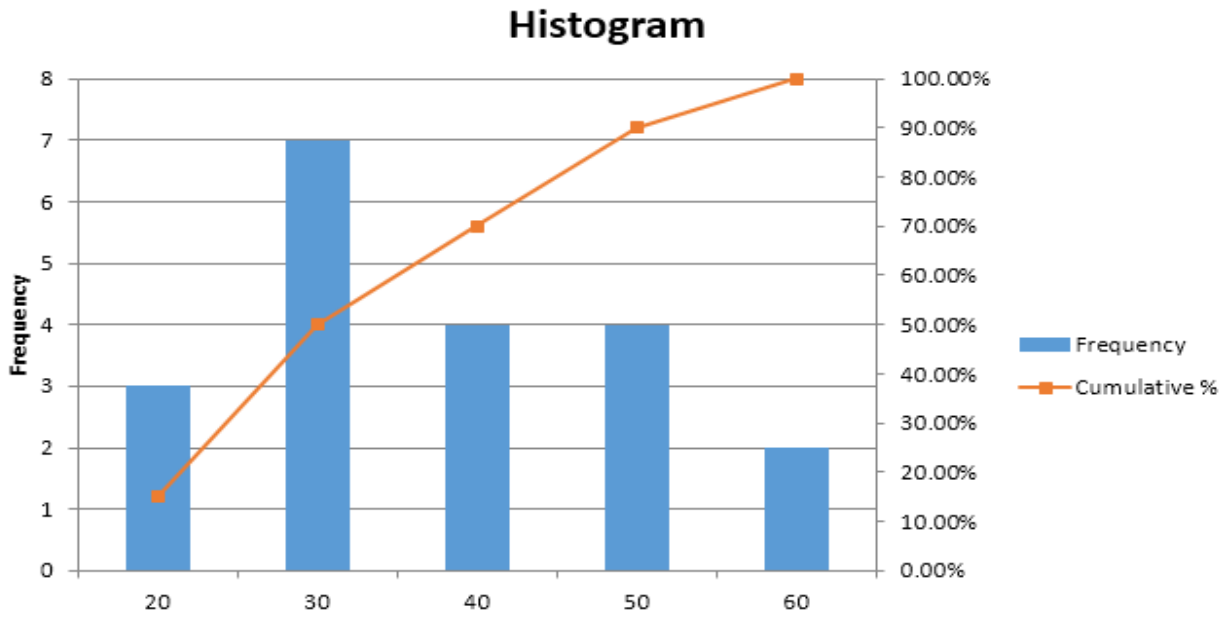

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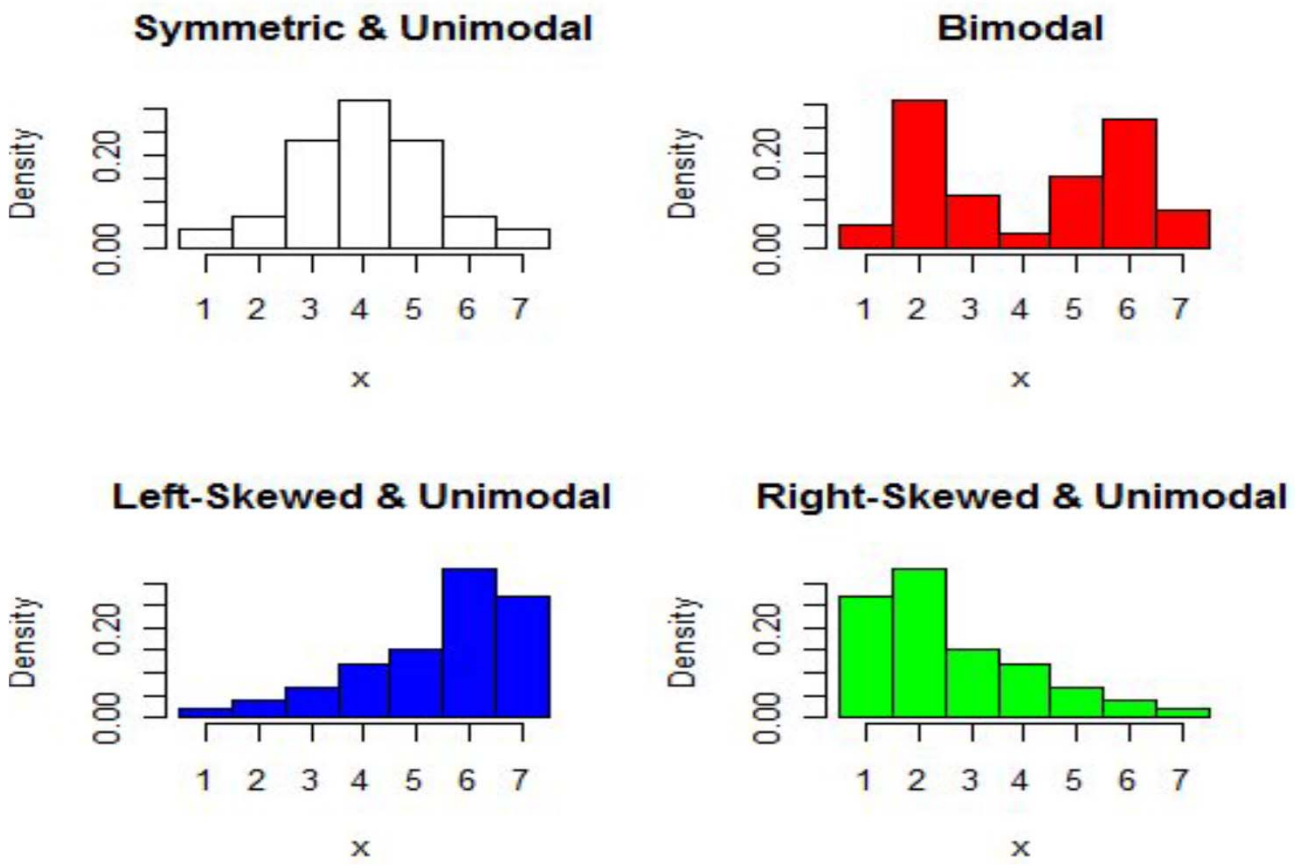
Graphing Data Distributions

Identifying Distribution Shapes



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Graphing Data Distributions

Quartiles

$$
\begin{aligned}
Q2 &= \text{second quartile of the } x_i\text{'s} \\
&= \begin{cases} \frac{1}{2}\left(x_{(k)} + x_{(k+1)}\right), & \text{if } n = 2k, \\ x_{(k+1)}, & \text{if } n = 2k+1. \end{cases}
\end{aligned}
$$

Note that $Q2$ is also referred to as the median of the x_i's.

The first quartile, denoted $Q1$, *may be defined* as the median of x_i values less than or equal to $Q2$.

The third quartile, denoted $Q3$, *may be defined* as the median of x_i values greater than or equal to $Q2$.

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Graphing Data Distributions

Survey of travelling time to school was conducted on 25 students

12	19	3	13	32
33	25	8	15	30
19	19	4	5	24
21	32	17	10	2
35	11	18	10	9

0	2 3 4 5 8 9
1	0 0 1 2 3 5 7 8 9 9 9
2	1 4 5
3	0 2 2 3 5

Q1=9.5 , Q2=17, Q3=19, Interquartile Range (IQR)=9.5

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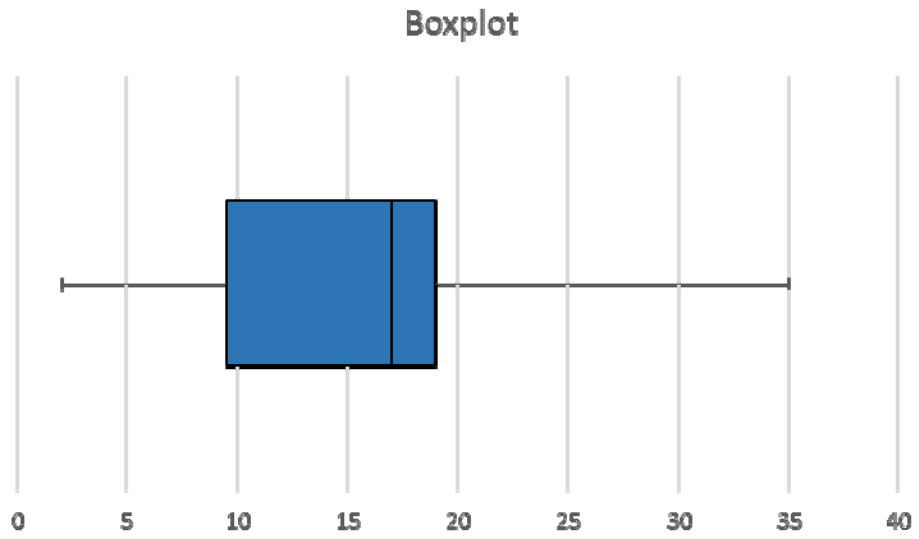
Graphing Data Distributions

Boxplot

Q1=9.5 , Q2=17, Q3=19, Interquartile Range (IQR)=9.5, min=2, max=35, lower limit = Q1-1.5 x IQR, upper limit = Q3 + 1.5 x IQR



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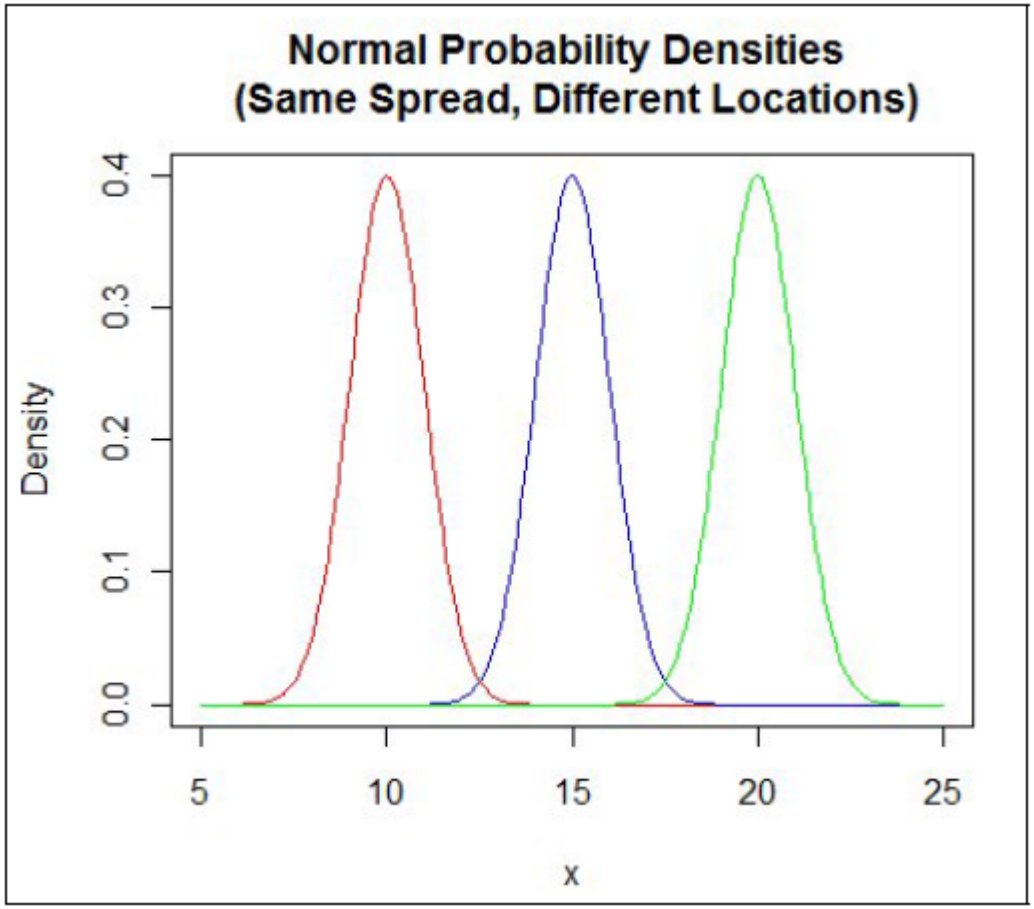
Measures of dispersion



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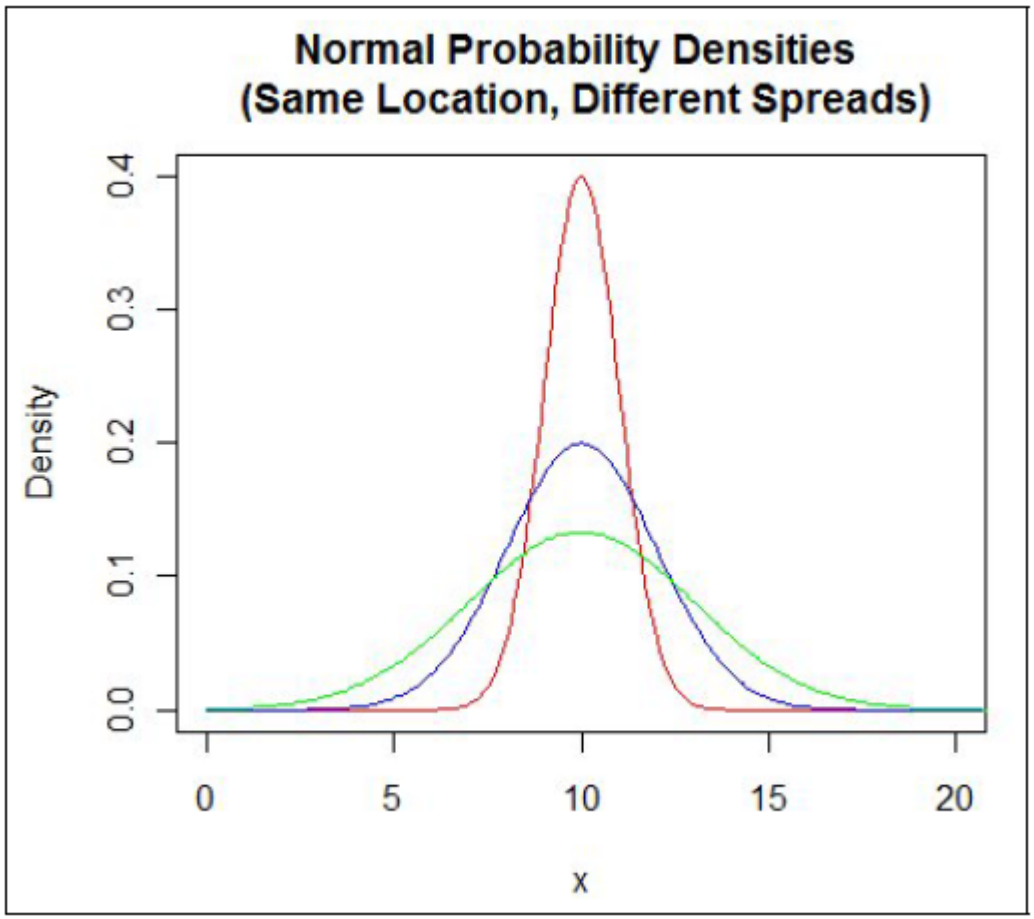
Measures of dispersion



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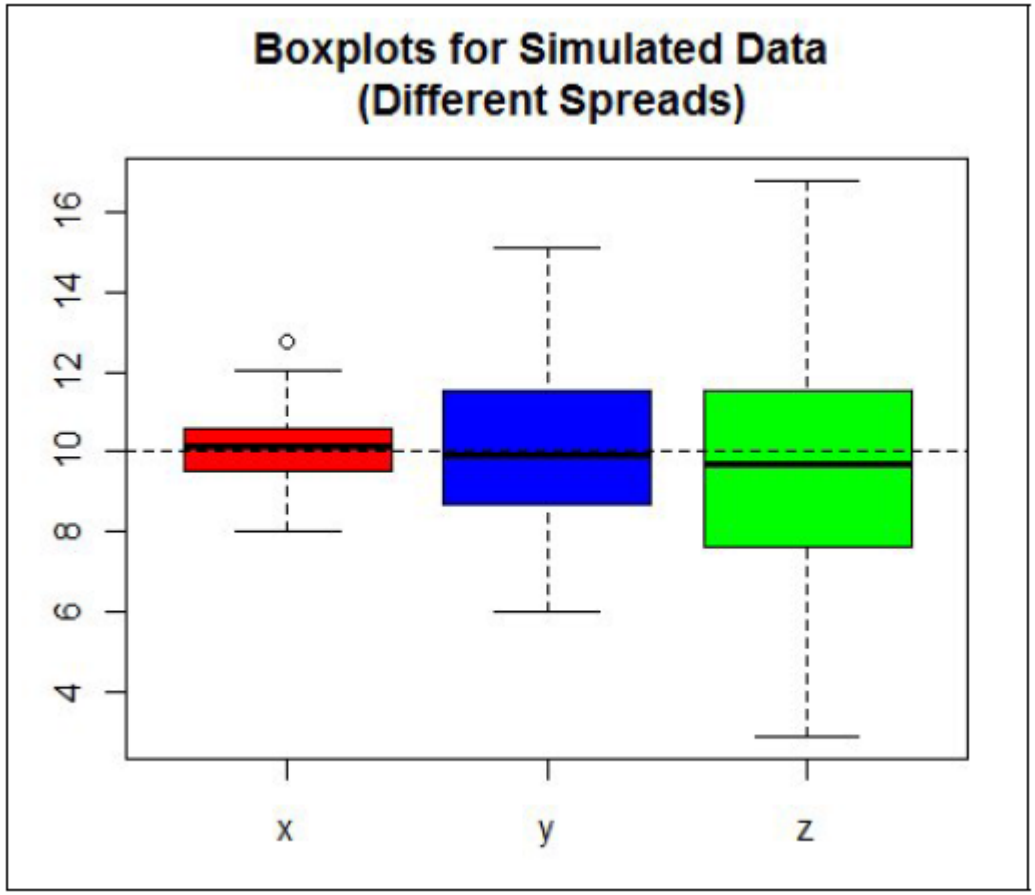
Measures of dispersion



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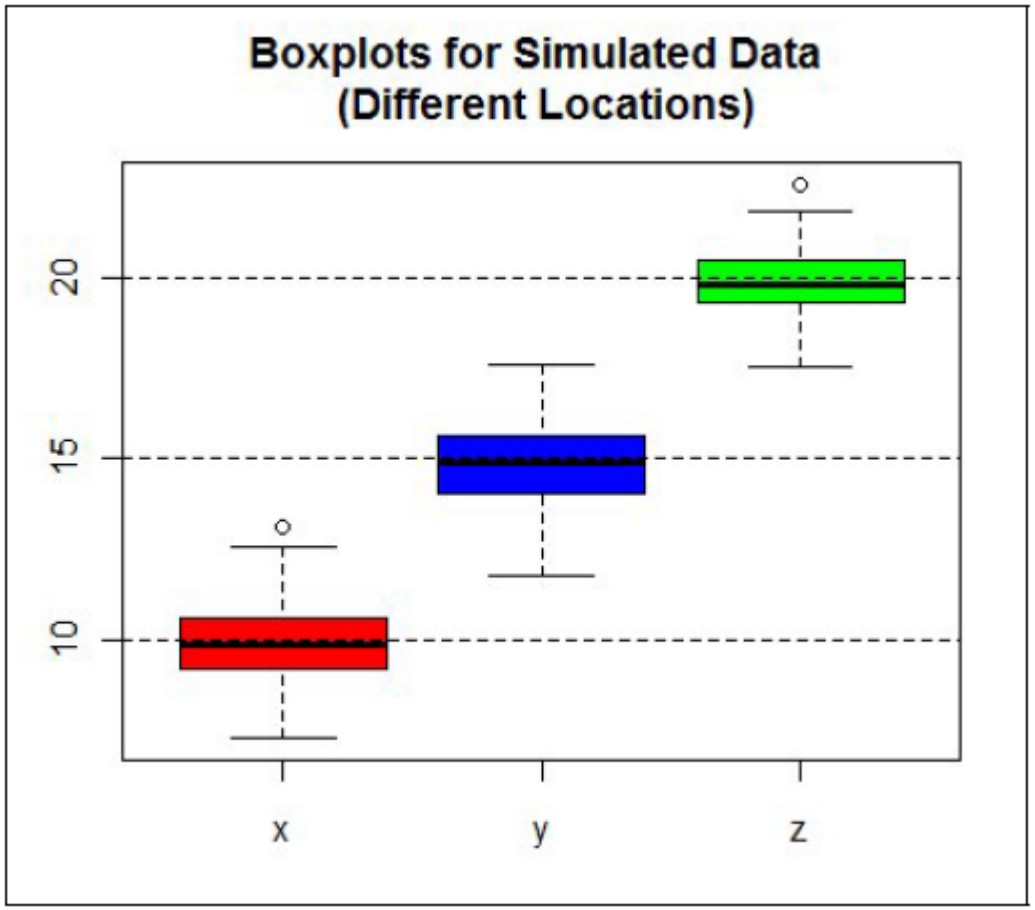
Measures of dispersion



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Measures of Central Tendency

- *Mean*

$$\text{mean} = \frac{1}{n}\sum_{i=1}^{n} x_i = \overline{x}, \text{ say.}$$

- *Median*

$$\text{median} = \begin{cases} \frac{1}{2}\left(x_{(k)} + x_{(k+1)}\right), & \text{if } n = 2k, \\ x_{(k+1)}, & \text{if } n = 2k+1. \end{cases}$$

- *Mode*

$$\text{mode} = \text{data value with highest frequency.}$$

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Measures of Central Tendency

0	2 3 4 5 8 9
1	0 0 1 2 3 5 7 8 9 9 9
2	1 4 5
3	0 2 2 3 5

Median=17, Mean = 17.04, Mode = 19

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Measures of Central Tendency

Advantages & Disadvantages

Feature	Mean	Median	Mode
Always Exists?	Y	Y	N
Always Unique?	Y	N	N
Not Affected by Outliers?	N	Y	Y
Further Analysis Potential?	Y	N	N

Note

- Use a robust (i.e., resistant) measure of central tendency when outlying values (assuming these are valid) are present.
- The trimmed mean is an example of a robust measure of location

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Measures of Dispersion

- *Range*

$$\text{range} = x_{(n)} - x_{(1)}$$

- *Interquartile Range*

$$\text{IQR} = \text{Third Quartile} - \text{First Quartile}$$

- *Variance*

$$\text{variance} = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x})^2$$

- *Standard Deviation*

$$\text{standard deviation} = \sqrt{\frac{1}{n-1}\left(\sum_{i=1}^{n} x_i^2 - n\overline{x}^2\right)}$$

A relative measure of dispersion is

$$\text{coefficient of variation} = \frac{\text{standard deviation}}{\text{mean}}.$$

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Measures of Dispersion

0	2 3 4 5 8 9
1	0 0 1 2 3 5 7 8 9 9 9
2	1 4 5
3	0 2 2 3 5

Range=33, Interquartile Range (IQR)=9.5

$$\text{Variance} = \frac{(2^2+2^2+\cdots+35^2) - 25*17.4^2}{25-1} \approx 87.04$$

$$\text{Standard Deviation} \approx \sqrt{87.04} \approx 9.33$$

$$\text{Coefficient of variation} = \frac{9.33}{17.4} \approx 0.54$$

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Measures of Dispersion

Advantages & Disadvantages

Feature	R	V	SD	IQR	CV
Always Exists?	Y	Y	Y	Y	Y
Always Unique?	Y	N	N	N	N
Not Affected by Outliers?	N	N	N	Y	N
Absolute Measure?	Y	Y	Y	Y	N
Same Units?	Y	N	Y	Y	N

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Mean & Variance for Grouped Data

Group data refers to data in a frequency distribution

- Let

$$\begin{aligned} m_i &= \text{mid-point of } i\text{-th class}, \\ n_i &= \text{frequency of } i\text{-th class}, \\ k &= \text{number of classes}, \\ n &= \text{total frequency}. \end{aligned}$$

- The grouped data mean is

$$\overline{x}_g = \sum_{i=1}^{k} m_i \times \frac{n_i}{n} = \frac{1}{n} \sum_{i=1}^{k} m_i n_i.$$

- The grouped data variance is

$$s_g^2 = \sum_{i=1}^{k} m_i^2 \times \frac{n_i}{n} - \overline{x}_g^2 = \frac{1}{n} \sum_{i=1}^{k} m_i^2 n_i - \overline{x}_g^2.$$

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Mean & Variance for Grouped Data

Class	ni	mi	ni*mi	ni* mi^2
(10,20]	3	15	45	675
(20,30]	7	25	175	4375
(30,40]	4	35	140	4900
(40,50]	4	45	180	8100
(50,60]	2	55	110	6050
Sum	20		650	24100

$$\bar{x}_g = \frac{650}{20} = 32.5$$

$$S_g{}^2 = \frac{24100}{20} - 32.5^2 = 148.75$$

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Summation Notation

Summation Notation

Given numerical values $x_1, \ldots, x_n$, we have:

- $\sum_{i=1}^{n} x_i = x_1 + x_2 + \cdots + x_n$
- $\sum_{i=1}^{n} (ax_i + b) = (ax_1 + b) + \cdots + (ax_n + b) = a\sum_{i=1}^{n} x_i + nb$

Example

If x_i's are given by $1.75, 2.25, 2.25, 2.25, 1.75, 2.00, 1.50$, we have

$$\sum_{i=1}^{7} x_i = 13.75 \text{ and } \sum_{i=1}^{7} x_i^2 = 1.75^2 + \cdots + 1.50^2 = 27.5625.$$

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Recap

Module 1: Introduction
Tabulating and graphing data distributions
Measures of dispersion
Measures of central tendency
Mean and variance for grouped/sample data
Summation Notation
Recap

Next Module:
Module 2: Discrete Distribution Problems

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