Question 1 (a) $\binom{51}{4}$

(b) $\binom{52}{5}\binom{47}{5}\binom{42}{5}\binom{37}{5}$

Question 2 (a) $\binom{20}{3}\binom{15}{2}$

(b) $\binom{16}{12}$

(c) $\binom{8}{2}$

Question 3 (i) $a_n = 2^{n-2} + a_{n-1}$

(ii) $a_1 = 0, a_2 = 1$

(iii) $a_n = 2^n - 1$

Question 4 $\binom{13}{5}$

Question 5

1. No
2. Yes
3. True
4. False
5. False
6. $n - k$
7. $\binom{n}{2}$
8. A concatenation of two C_3 between any two vertices from two different C_3.
9. K_4
10. $K_{n,2}$ is always planar.

Question 6

1. *Proof.* Suppose not, $\sum_{i=1}^{n} \deg(v_i) \geq 2n-1$. Since the sum of degrees are twice of the size of the edge set, we have

$$|E| \geq \frac{n-1}{2}.$$

Moreover, $|E|$ is an integer, so $|E| \geq n$. But a tree with n vertices can have at most $n-1$ edges. Thus it is impossible. □

2. *Proof.* Suppose $e_1 = (v_0, v_1)$ and $e_2 = (v_2, v_3)$. Since G is connected, there is a path from v_0 to v_2, denoted by P. If v_3 is not in P, we add e_2 to the tail of P, i.e. the path is $(v_0, \dots, v_2, v_3)$. Now e_2 is in P. Moreover, if e_1 is not in P, we add it to the head, i.e. P becomes $(v_1, v_0, \dots, v_2, v_3)$. In all cases a path desired is found. $\square$